Practice 8-1/8-2

1. A basket player has made 80% of his foul shots during the season. Assume the shots are independent, find the probability that in tonight’s game he:

   A) Misses for the first time on his fifth attempt.

   \[ p = 0.80 \text{ makes shot} \]
   \[ q = 0.20 \text{ misses shot} \]
   \[ X = \text{number of shots until he misses it} \]
   \[ P(X = 5) = \text{Geometpdf}(0.20, 5) = 0.08192 \]

   B) Makes his first basket on his fourth shot.

   \[ X = \text{number of shots until he makes it} \]
   \[ P(x = 4) = \text{Geometpdf}(0.80, 4) = 0.0064 \]

   C) Makes his first basket on one of his first 3 shots.

   \[ P(X \leq 3) = P(X = 1) + P(X = 2) + P(X = 3) = \text{geometcdf}(0.80, 3) = 0.992 \]

2. For the basketball player in Question 1, what’s the expect number of shots until he misses?

   \[ X = \text{number of shots until he misses it} \]

   \[ \mu = \frac{1}{p} = \frac{1}{0.20} = 5 \]

3. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome described below.

   A) The first lefty is the fifth person chosen.

   \[ p = 0.13 \text{ are left handed where } X = \text{number of people until a lefty is found} \]
   \[ P(X = 5) = \text{Geometpdf}(0.13, 5) = 0.0745 \]

   B) There are some lefties among the 5 people.

   \[ P(X \leq 5) = \text{geometcdf}(0.13, 5) = 0.502 \]

   C) The first lefty is the second or third person.

   \[ P(X = 2 \text{ or } X = 3) = P(X = 2) + P(X = 3) \]
   \[ = \text{Geometpdf}(0.13, 2) + \text{Geometpdf}(0.13, 3) = 0.211 \]
D) There are exactly 3 lefties in the group.

\[ P(X = 3) = \binom{5}{3}(0.13)^3(0.87)^2 = 0.0166 \]

E) There are at least 3 lefties in the group.

\[ P(X \geq 3) = 1 - P(X \leq 2) \\
= 1 - \binom{5}{2}(0.13)^2(0.87)^3 = 0.0179 \]

F) There are no more than 3 lefties in the group.

\[ P(X \leq 3) = \binom{5}{3}(0.13)^3(0.87)^2 = 0.9987 \]

4. Consider our group of 5 people from Question 3.

A) How many lefties do you expect?

\[ \mu = np = 5(0.13) = 0.65 \]

B) With what standard deviation?

\[ \sigma = \sqrt{npq} = \sqrt{5(0.13)(0.87)} = 0.75 \]

C) If we keep picking people until we find a lefty, how long do you expect it will take?

\[ \mu = \frac{1}{p} = \frac{1}{0.13} = 7.69 \text{ picks} \]

5. A newly hired telemarketer is told he will make a sale on about 12% of his phone calls. The first week he called about 200 people, but only made 10 sales. Should he suspect he was mislead about the true success rate? Explain

\[ x = \text{make sales}, \ p = 0.12, \ n = 200 \]
\[ \mu = np = 200(0.12) = 24 \]
\[ \sigma = \sqrt{npq} = \sqrt{200(0.12)(0.88)} = 4.595 = 4.60 \]

The mean number of sales should be 24 with standard deviation 4.60. Ten sales would be 3 standard deviation below the mean \(24 - 3(4.60) = 10.2\)

6. An airline, believing that 5% of passengers fail to show up for flights, overbooks (sells more tickets than there are seats). Suppose a plane will hold 265 passengers, and the airline sells 275 seats. What’s the probability the airline will not have enough seats so someone gets bumped?

\[ x = \text{will not have enough seats}, \ n = 275, \ p = 0.95, \ q = 0.05 \]
\[ P(X \geq 266) = 1 - P(X \leq 265) = 1 - \binom{275}{265}(0.95)(0.05) = 0.1155 = 0.116 \]
Or using normal approximation – satisfies \( np \geq 10 \) and \( nq \geq 10 \)

\[
\mu = np = 275(0.95) = 261.25 \\
\sigma = \sqrt{npq} = \sqrt{275(0.95)(0.05)} = 13.614
\]

\[
P(X \geq 266) = \text{normalcdf}(266, E99, 261.25, 13.614) = 0.0943
\]

7. The Los Angeles Times (December 13, 1992) reported that what airlines passengers like to most on long flights is rest or sleep; in survey of 3697 passengers, almost 80% did so. Suppose that for particular route the actual percentage is exactly 80%, and consider randomly selecting six passengers. Then, \( x \), the number among the selected six who rested or slept, is a binomial random variable with \( n = 6, p = 0.80 \)

A) Calculate \( P(X = 4) \), and interpret this probability.

\[
x = \text{rested or slept}, n = 6, p = 0.80 \\
P(X = 4) = \text{binompdf}(6, 0.80, 4) = 0.24576
\]

B) Calculate \( P(X = 6) \), the probability that all six selected passengers rested or slept.

\[
P(X = 6) = \text{binompdf}(6, 0.80, 6) = 0.262144
\]

C) Determine \( P(X \geq 4) \).

\[
P(X \geq 4) = 1 - P(X \leq 3) = 1 - \text{binomcdf}(6, 0.80, 3) = 0.90112
\]

8. Thirty percent of all automobiles undergoing an emissions inspection at a certain inspection station fail the inspection.

A) Among 15 randomly selected cars, what is the probability that at most 5 fail the inspection?

\[
x = \text{failed inspections}, n = 15, p = .30 \\
P(X \leq 5) = \text{binomcdf}(15, 0.30, 5) = 0.7216 = 0.722
\]

B) Among 15 randomly selected cars, what is the probability that between 5 and 10 (inclusive) fail to pass inspection?

\[
P(5 \leq X \leq 10) = \text{binomcdf}(15, 0.30, 10) - \text{binomcdf}(15, 0.30, 4) = 0.4838 = 0.484
\]

C) Among 25 randomly selected cars, what is the mean value of the number that pass inspection, and what is the standard deviation of the number that pass inspection?

\[
\mu = np = 25(0.70) = 17.5 \\
\sigma = \sqrt{npq} = \sqrt{25(0.70)(0.30)} = 2.2913
\]
D) What is the probability among 25 randomly selected cars, the number that pass is within 1 standard deviation of the mean value?

\[
\text{Within 1 standard deviation } (\mu \pm \sigma = 17.5 \pm 2.293) = 19.79, 15.2 \\
P(20 \leq X \leq 15) = \text{binomcdf}(25, 0.70, 20) - \text{binomcdf}(25, 0.70, 15) = 0.617 \\
\text{Cannot not use a normal approximate because it fails } np \geq 10 \text{ and } nq \geq 10
\]

9. Sophie is a dog that loves to play catch. Unfortunately, she isn’t very good, and probability that she catches a ball is only 0.10. Let \( x \) be the number of tosses required until Sophie catches a ball.

A) Does \( x \) have a binomial or geometric distribution?

Geometric Distribution

B) What is the probability that it will take exactly two tosses for Sophie to catch a ball?

\( x = \text{number of tosses required to catch the ball, } p = .10 \) \\
P(X = 2) = \text{geometpdf}(0.10, 2) = 0.09

C) What is the probability that more than three tosses will be required?

\[
P(X \geq 3) = 1 - P(X \geq 3) = 1 - \text{geometcdf}(0.10, 3) = 0.729
\]

10. About 8% of males are colorblind. A researcher needs some colorblind subjects for an experiment, and begins checking potential subjects.

A) On average, how many men should the researcher expect to check to find one who is colorblind?

\[
\mu = \frac{1}{p} = \frac{1}{0.08} = 12.5
\]

B) What’s the probability that she won’t find anyone colorblind among the first 4 men she checks?

\( x = \text{colorblind subjects, } p = 0.08 \) \\
P(X > 4) = 1 - P(X \leq 4) = 1 - \text{geometcdf}(0.08, 4) = 0.716

C) What’s the probability that the first colorblind man found will be the sixth person checked?

\[
P(X = 6) = \text{geometpdf}(0.08, 6) = 0.0527
\]

D) What’s the probability that she finds someone who is colorblind before checking the tenth man?

\[
P(X < 10) = \text{geometcdf}(0.08, 9) = 0.5278 = 0.528
\]